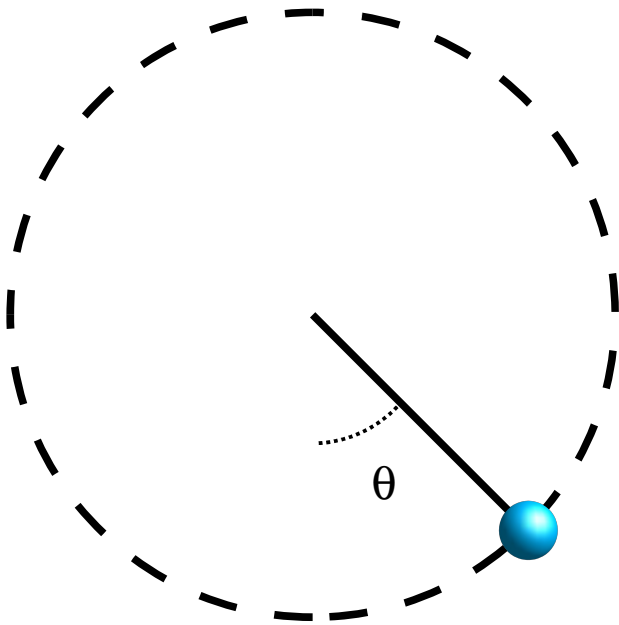


# Resonance effects in the atom optics kicked rotor

Maarten Hoogerland  
University of Auckland



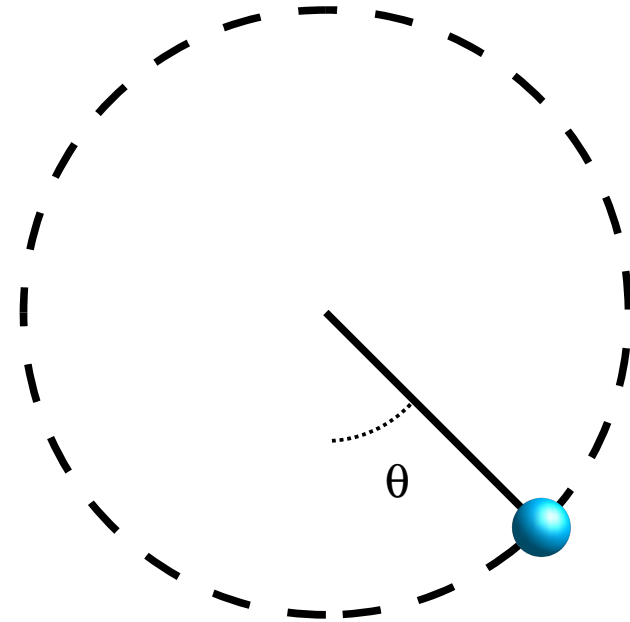
Jean-Anne Currivan  
Arif Ullah

# Outline

- Delta-kicked rotors
- Fractional resonances
- Momentum dependence
  - Time reversal
  - The future?

# Delta-kicked rotor

- Force switched on periodically
- Classically chaotic system
- Quantum correlations lead to reduced energy growth



$$V = -V_0 \cos \theta \sum_n \delta(t - nT)$$

# Talbot effect

- Well known optical gratings are reformed after Talbot distance

- For atoms: temporal

- Free evolution between kicks:

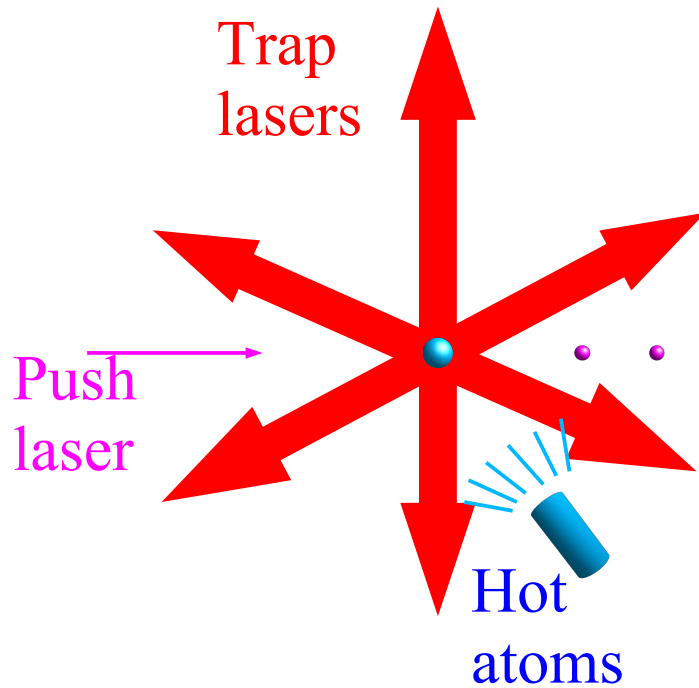
$$U_{free} = \exp(-i 4 \omega_r T)$$

- $U_{free} = 1$  For  $T = \frac{\pi}{2 \omega_r}$

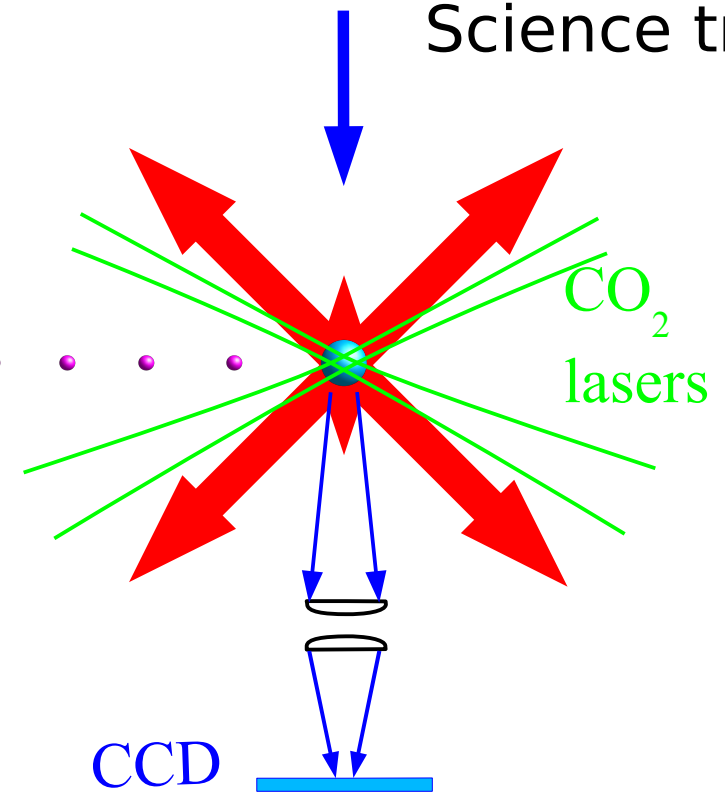
- This is the Quantum Resonance

# All Optical BEC Setup

Source trap



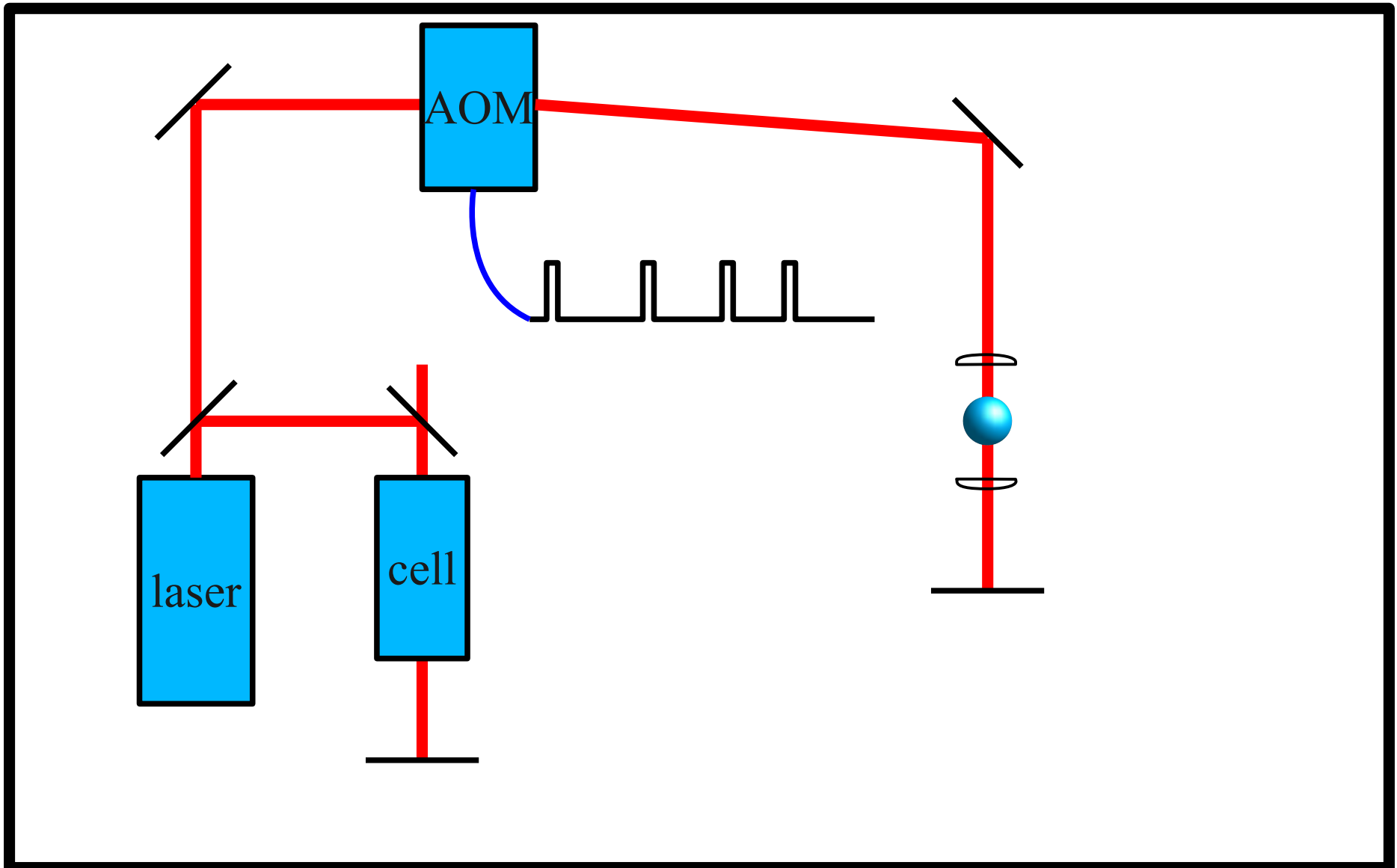
Science trap



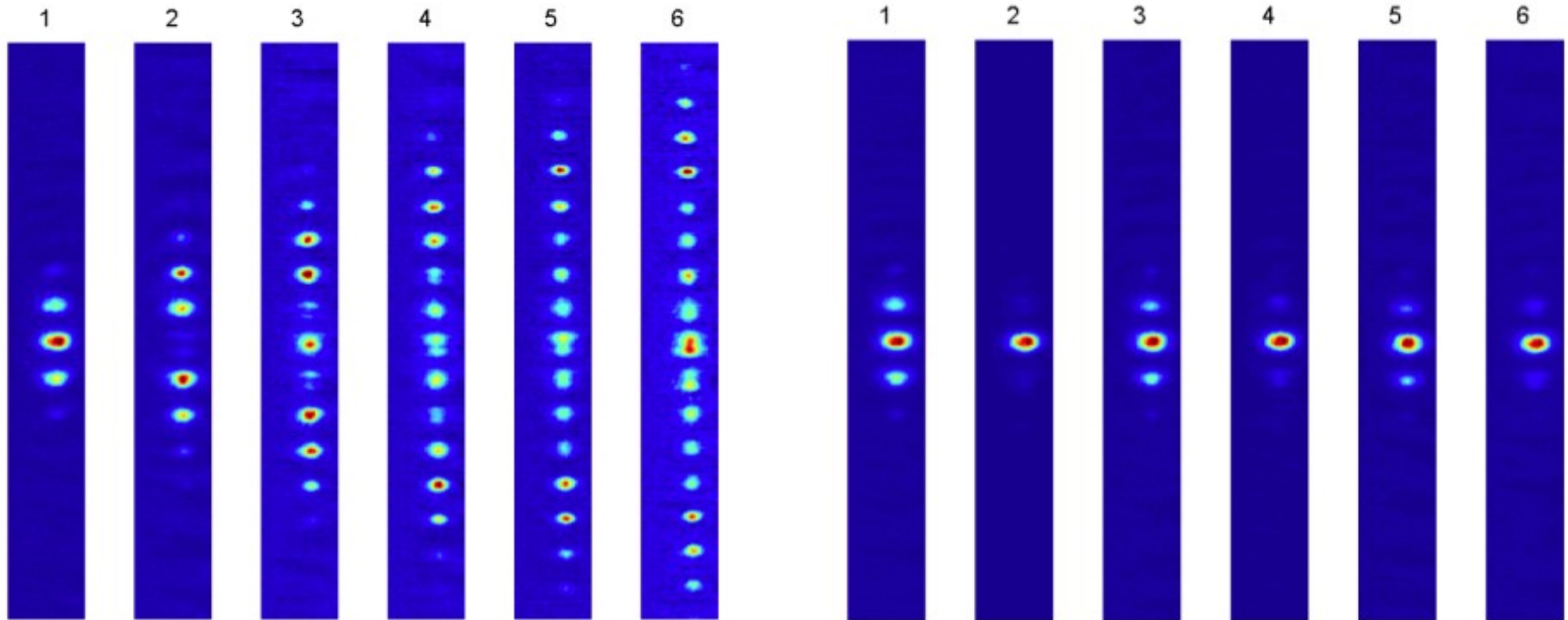
# Moving the lab



# Kicked rotor setup



# Talbot effect

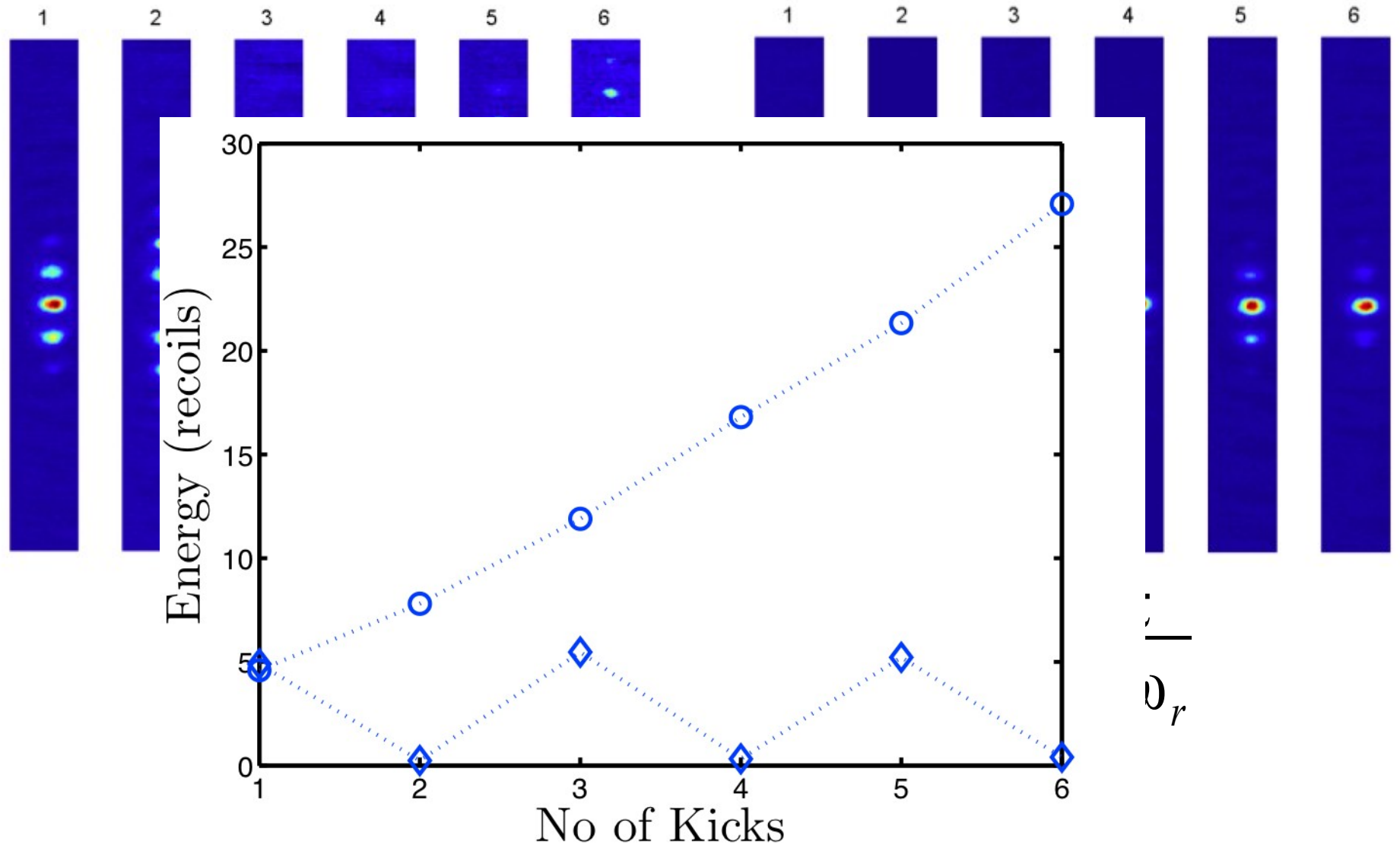


$$T = \frac{\pi}{2\omega_r}$$

$$T = \frac{\pi}{4\omega_r}$$



# Talbot effect



# Simulations

- Start with wave packet

$$\psi = C \exp\left(\frac{-x^2}{2\sigma^2}\right) \exp(ik_a x)$$

- Time evolution

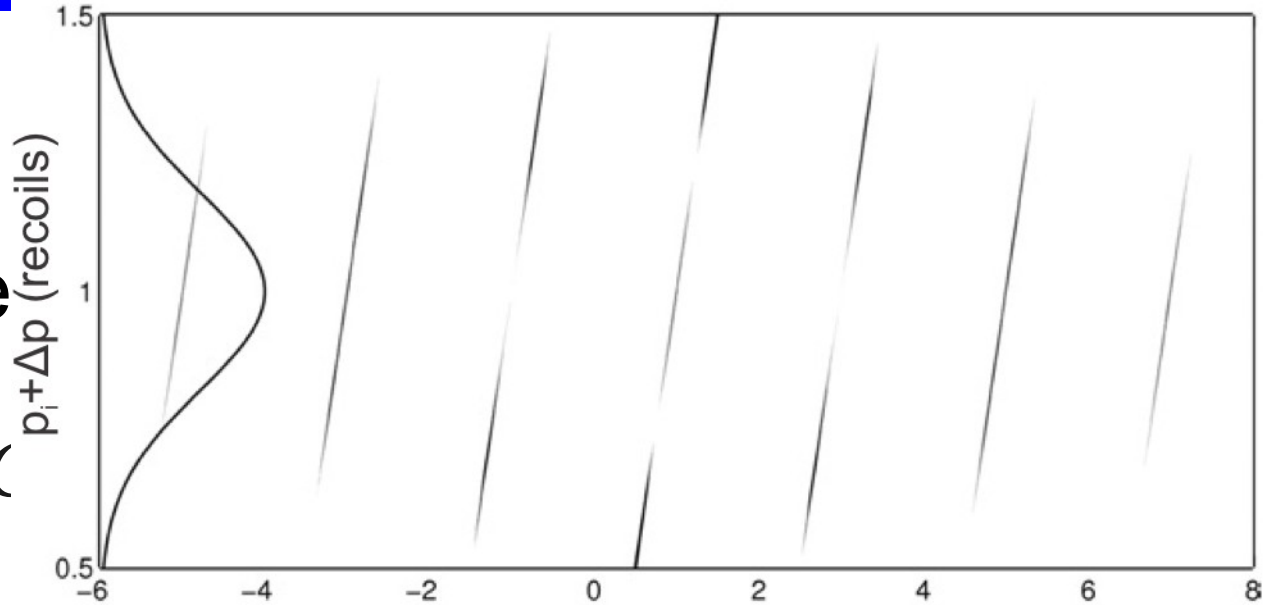
$$H = \frac{p^2}{2m} + V_0 \cos(2k_l x) \sum_n f(t - nT)$$

- Integrate  $k_a$  over momentum distribution BEC

# Simulations

- Start with wave

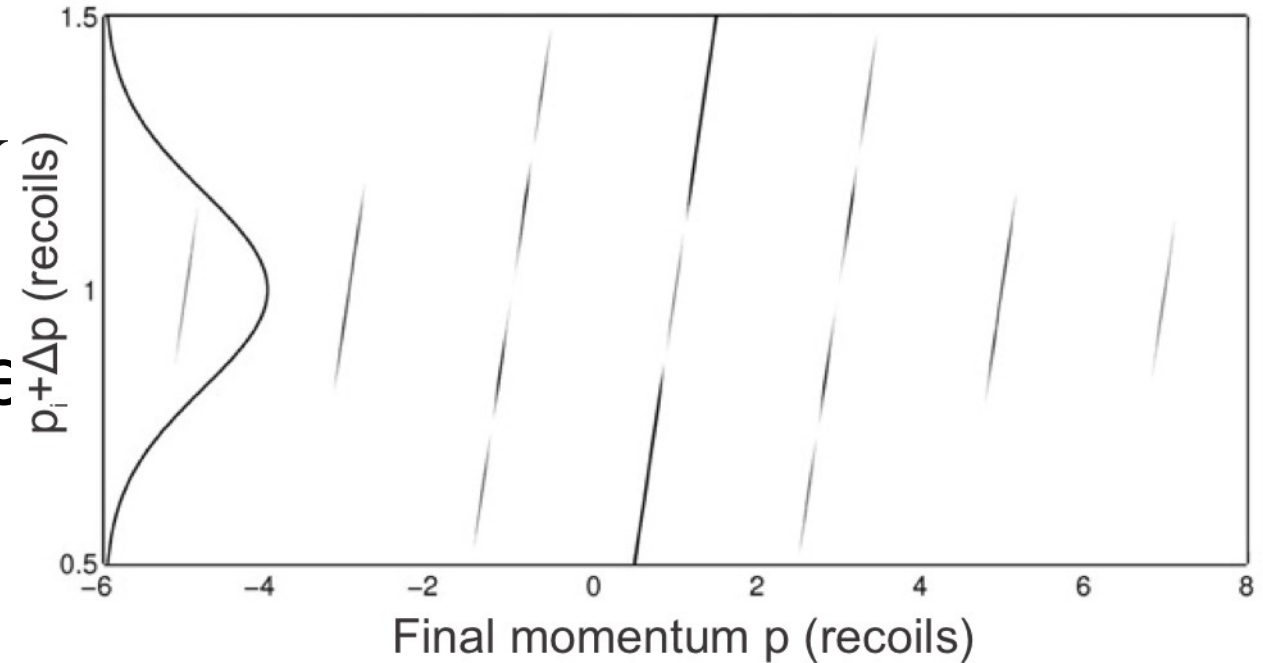
$$\psi = \left( \begin{array}{c} p_i + \Delta p \text{ (recoils)} \\ \end{array} \right)$$



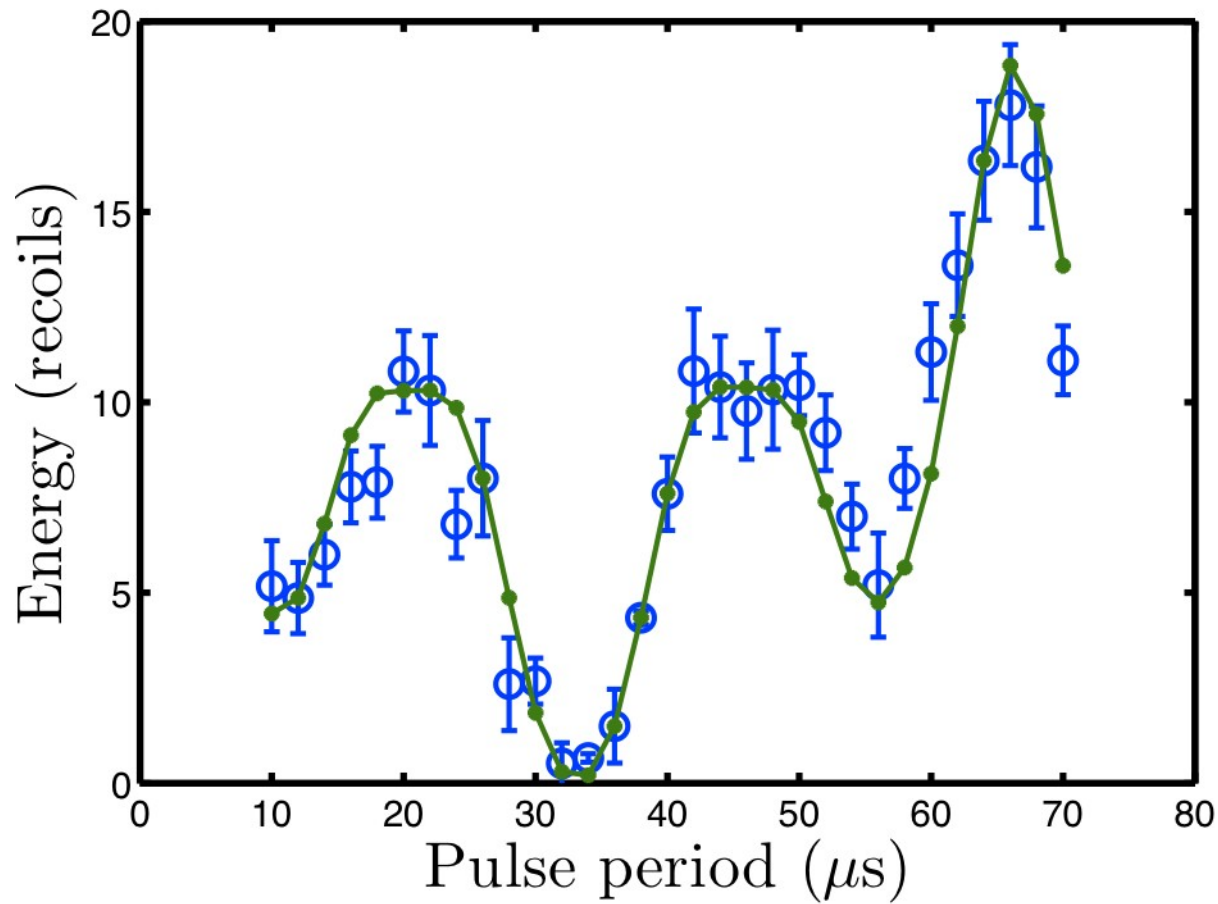
- Time evolution

$$H = \frac{p^2}{2m} + V$$

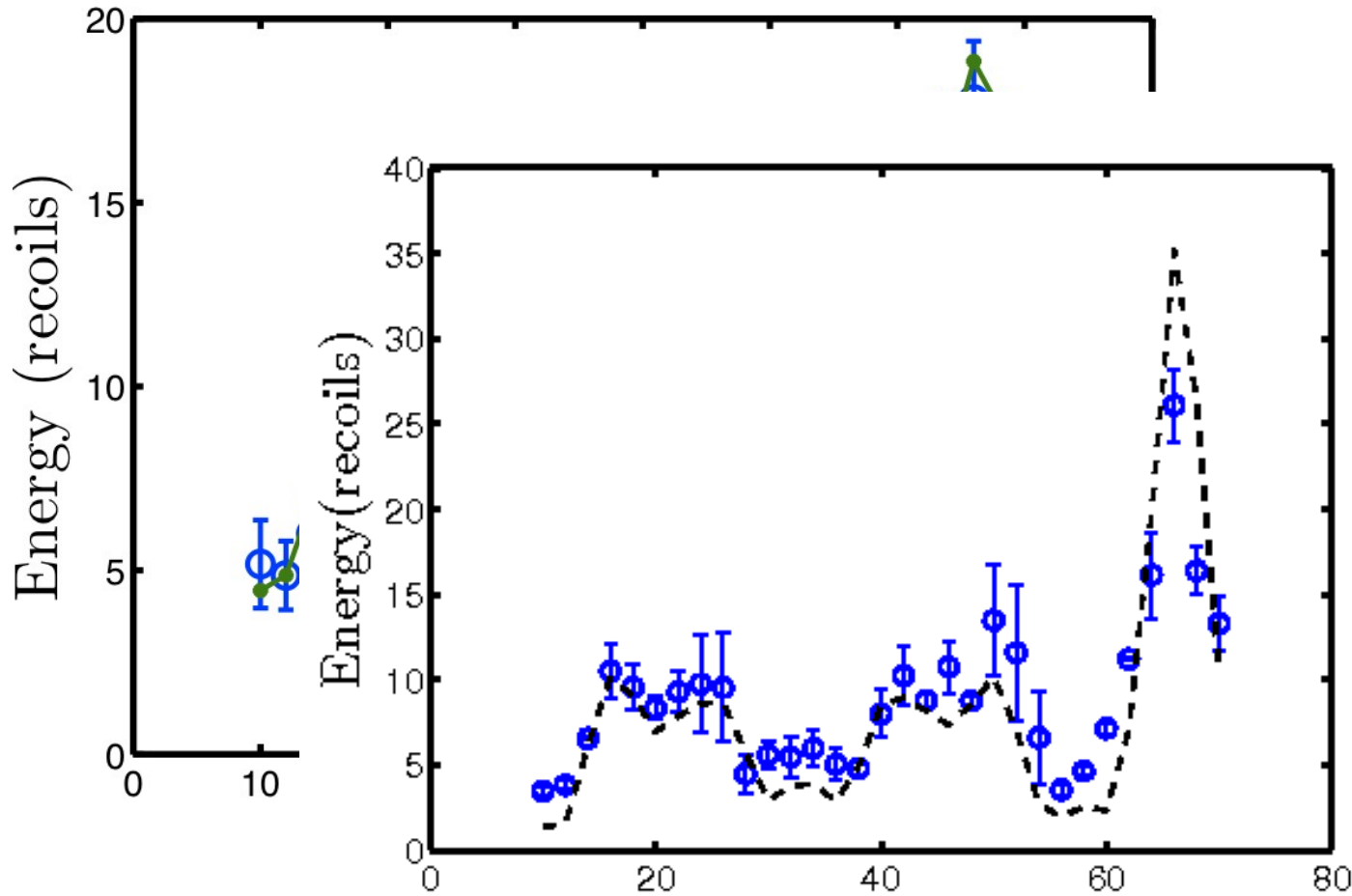
- Integrate  $k_a$  over



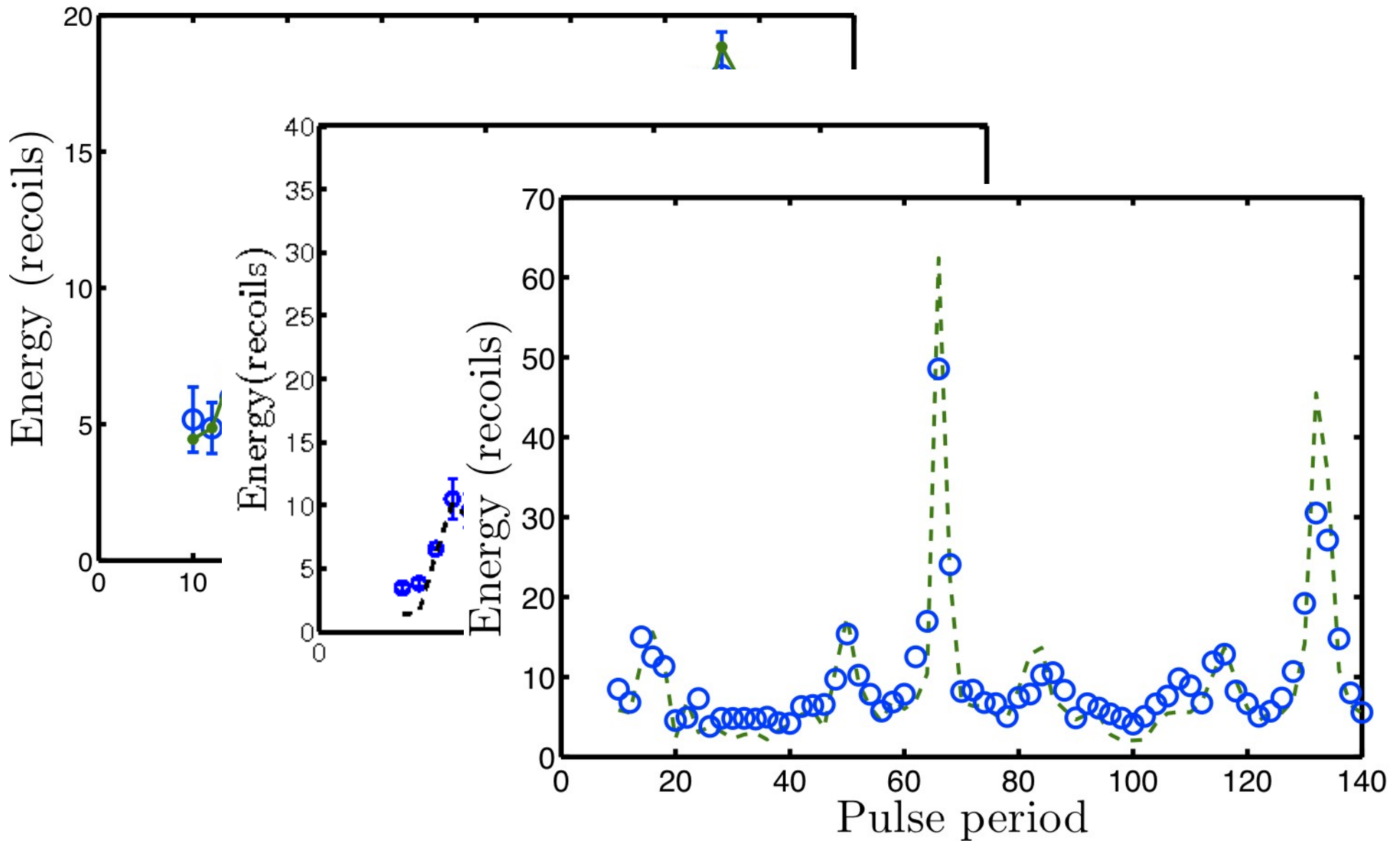
# Varying period



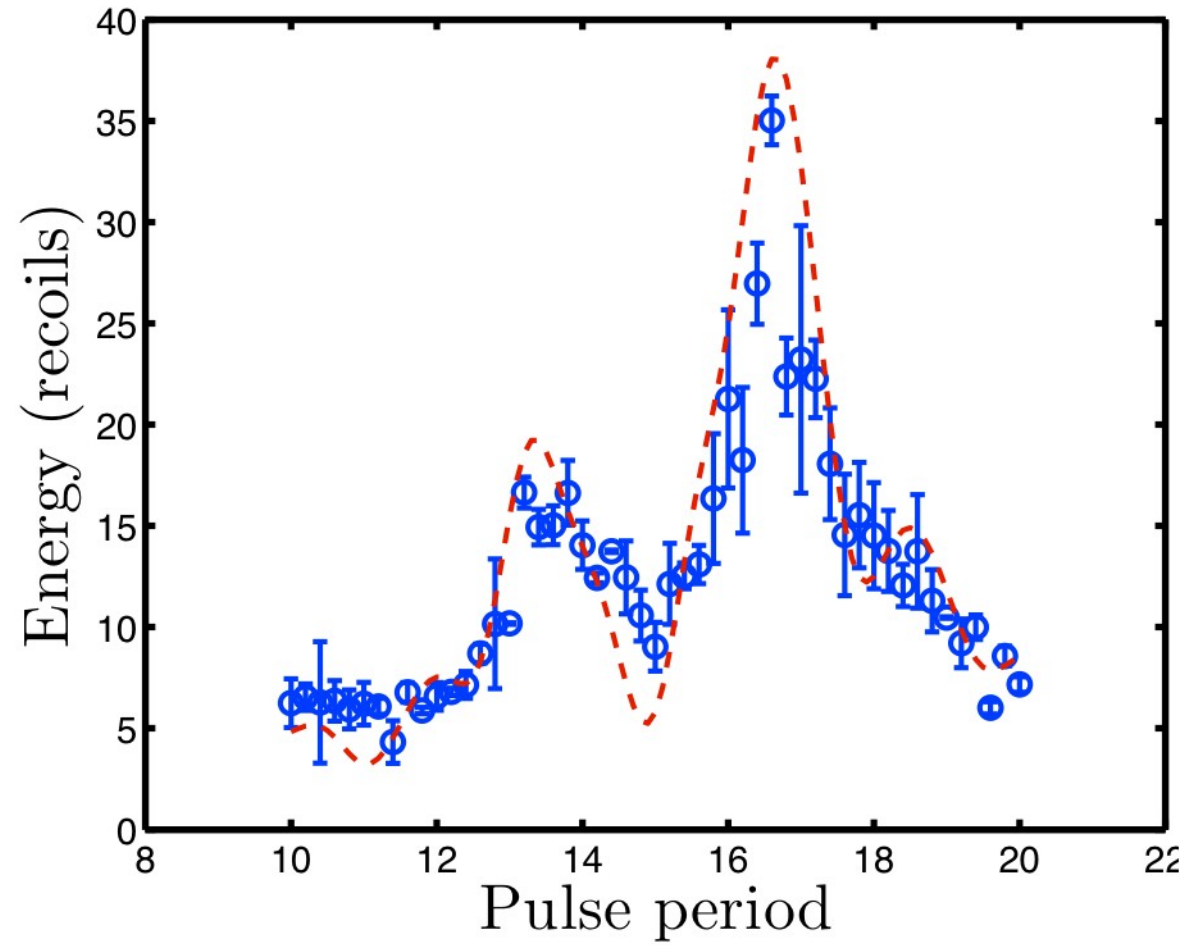
# Varying period



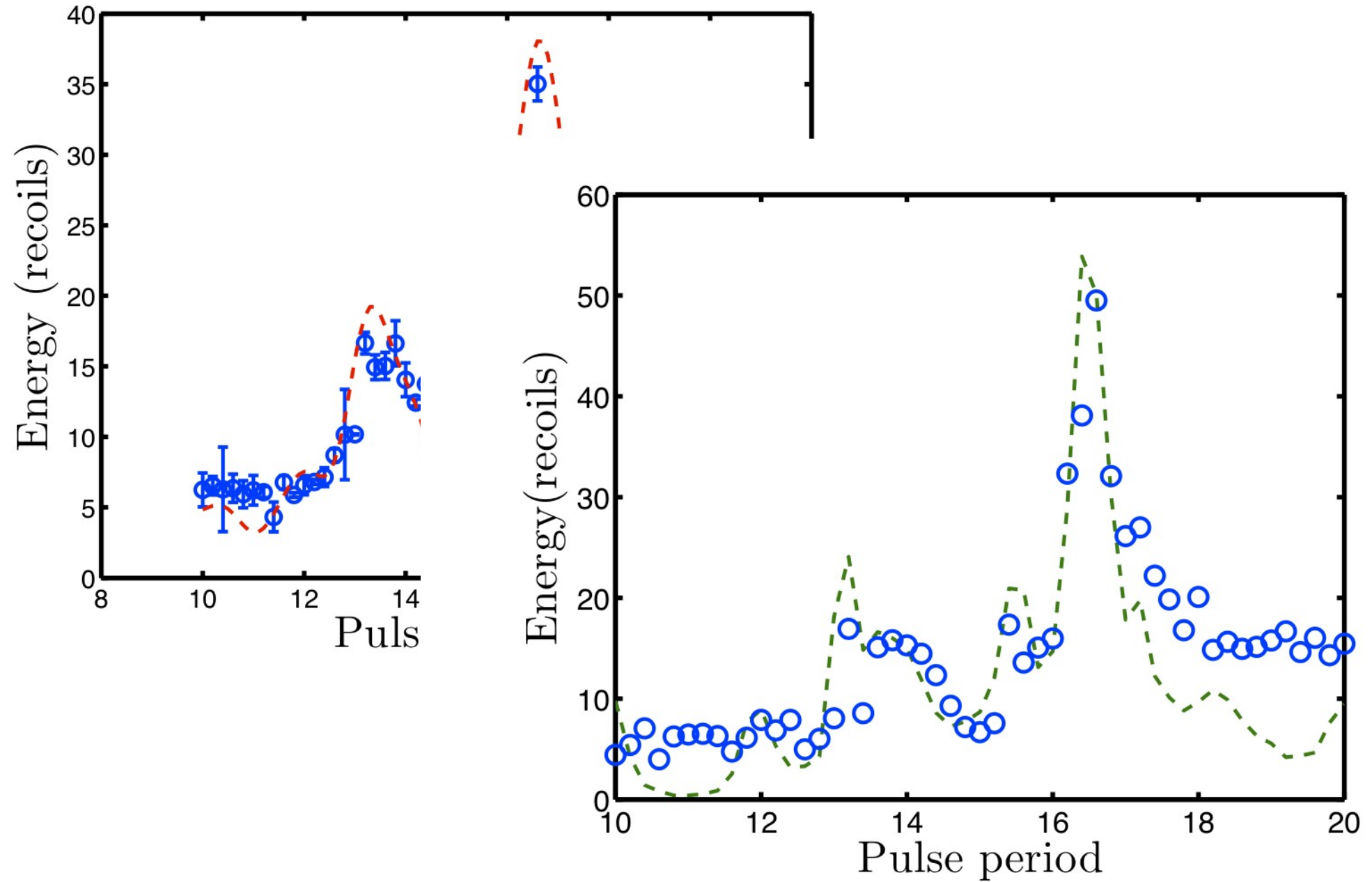
# Varying period (5 kicks)



# $T_T/4$ resonance

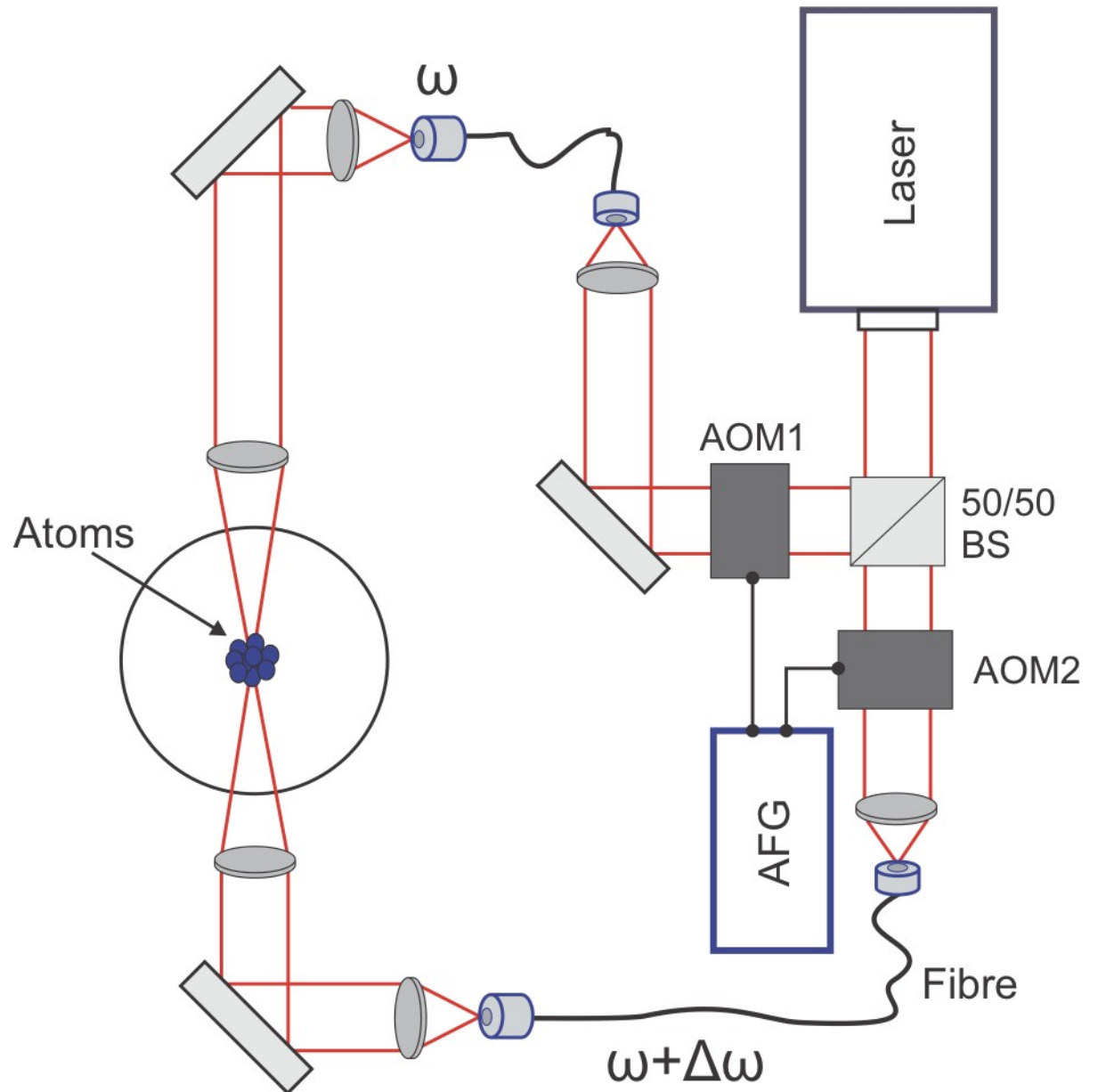


# $T_T/4$ resonance

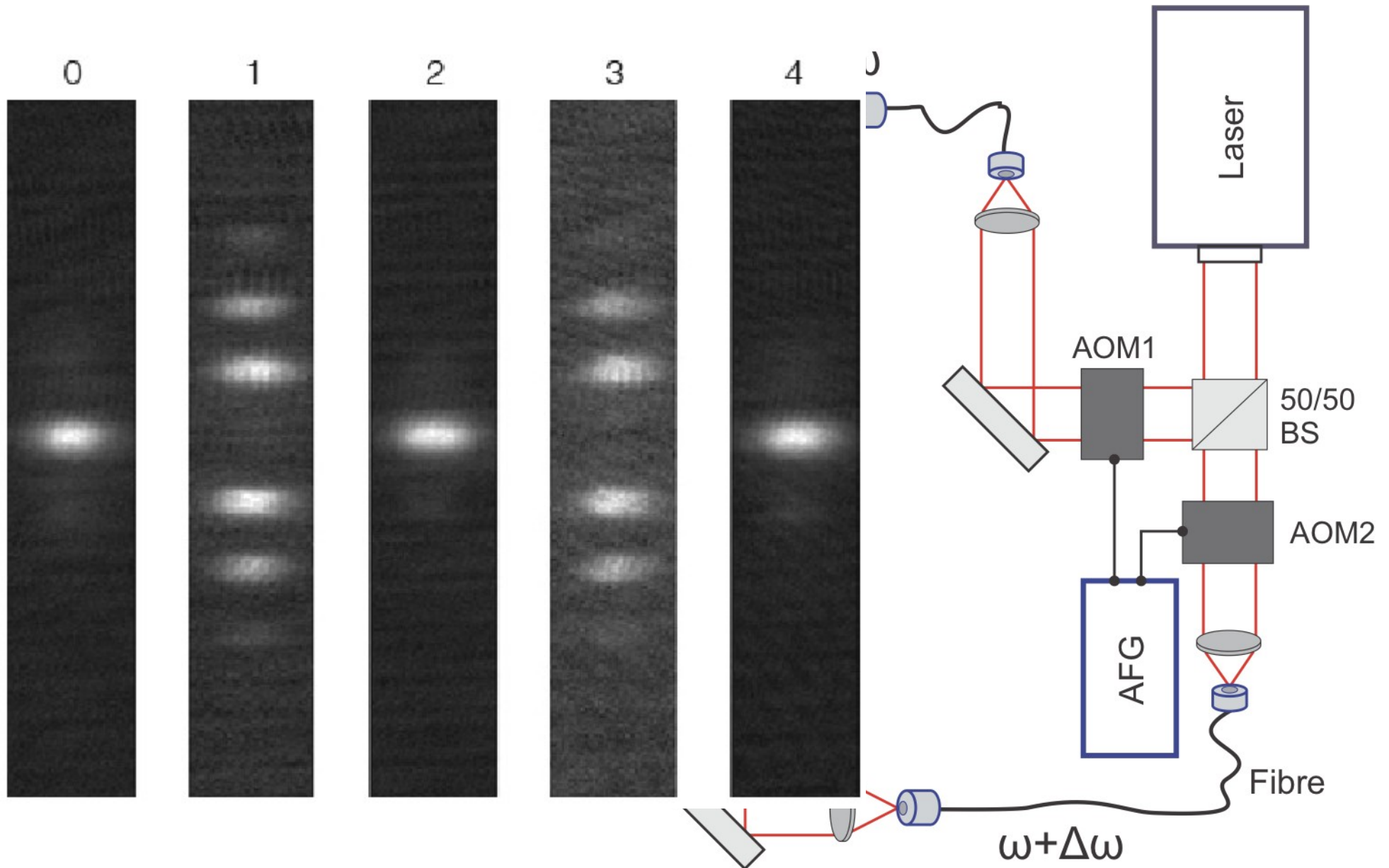




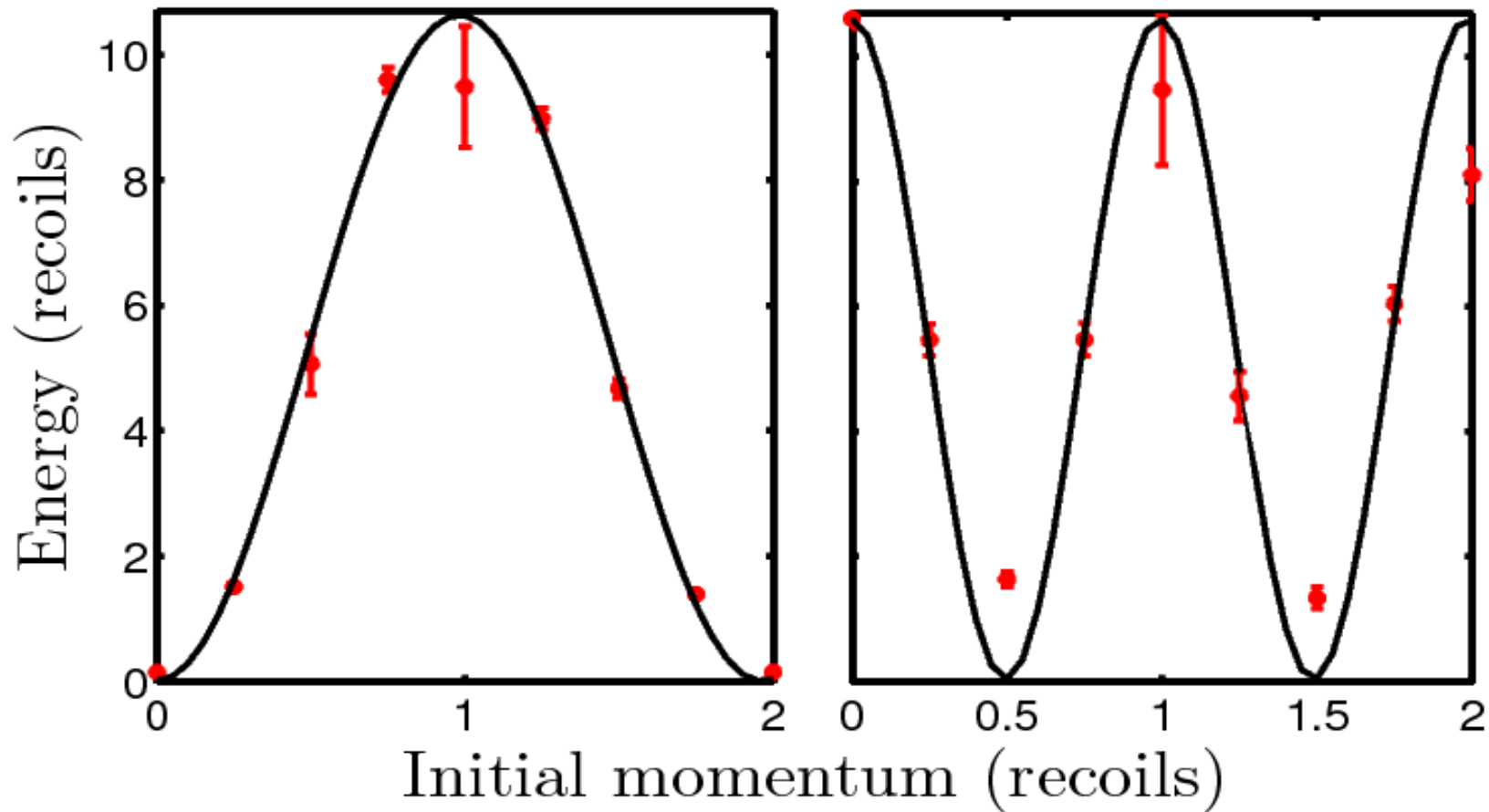
# Initial momentum



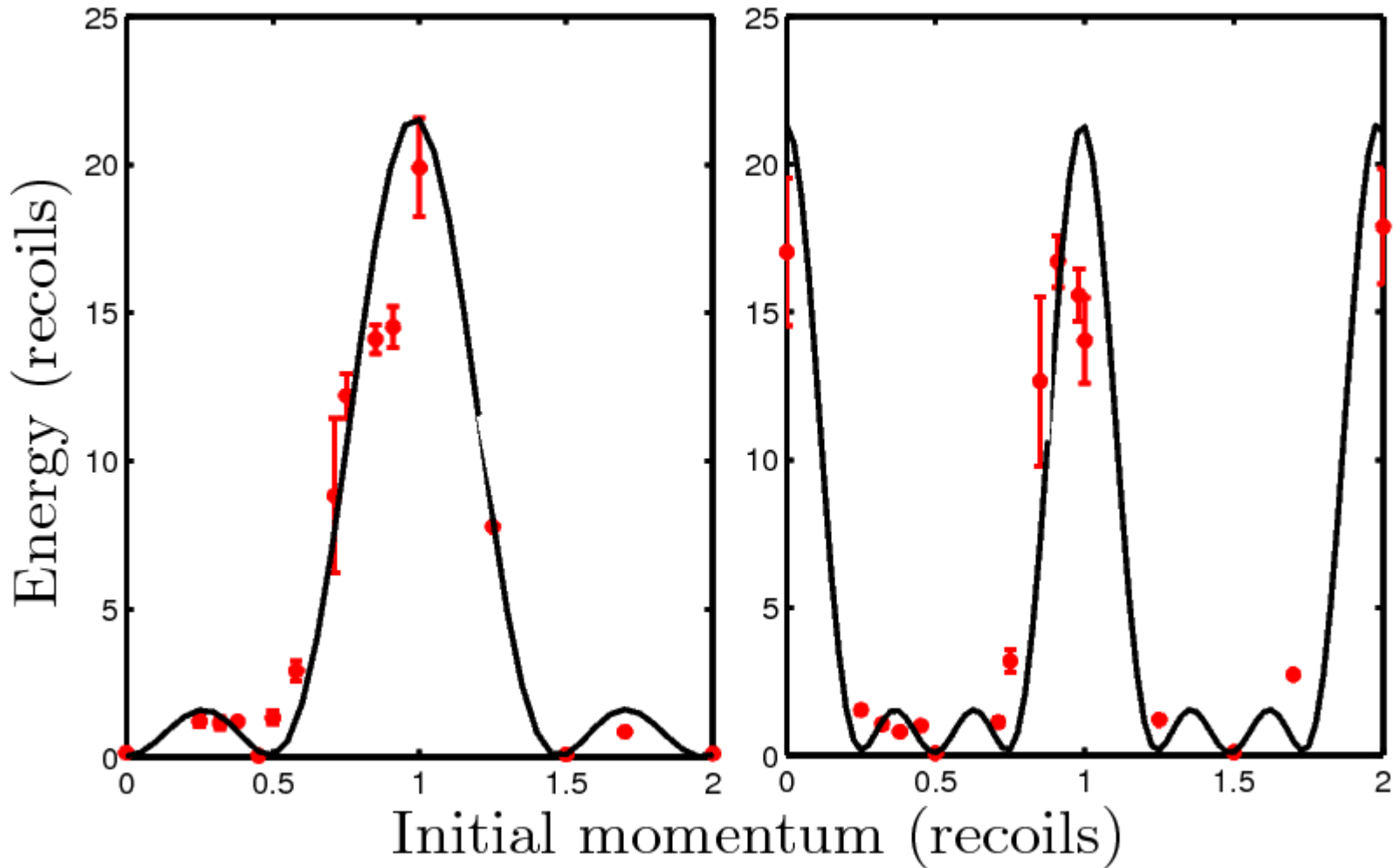
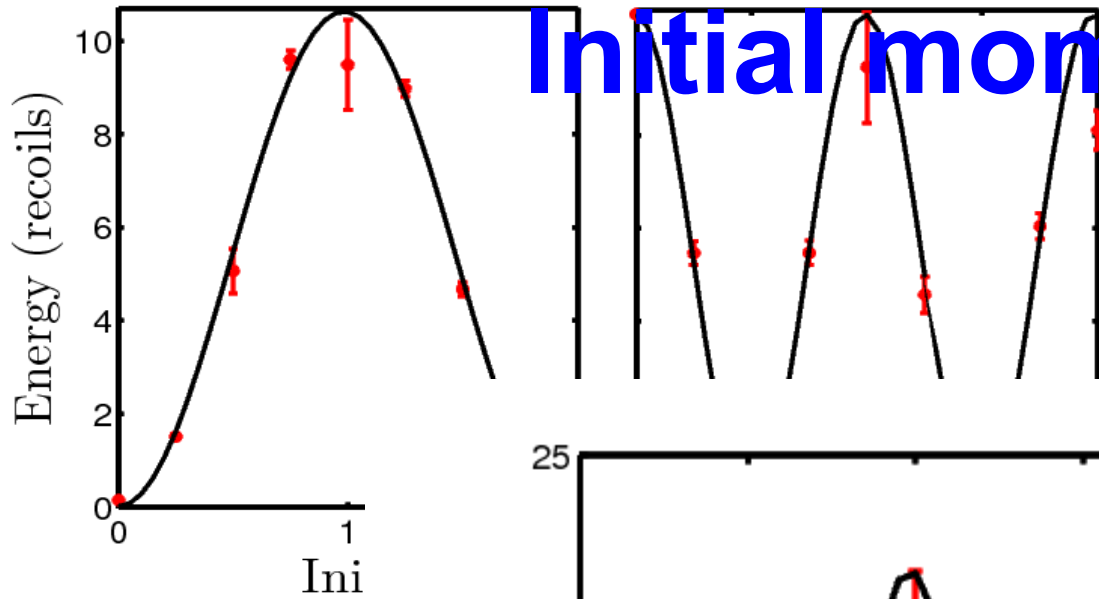
# Initial momentum



# Initial momentum



# Initial momentum

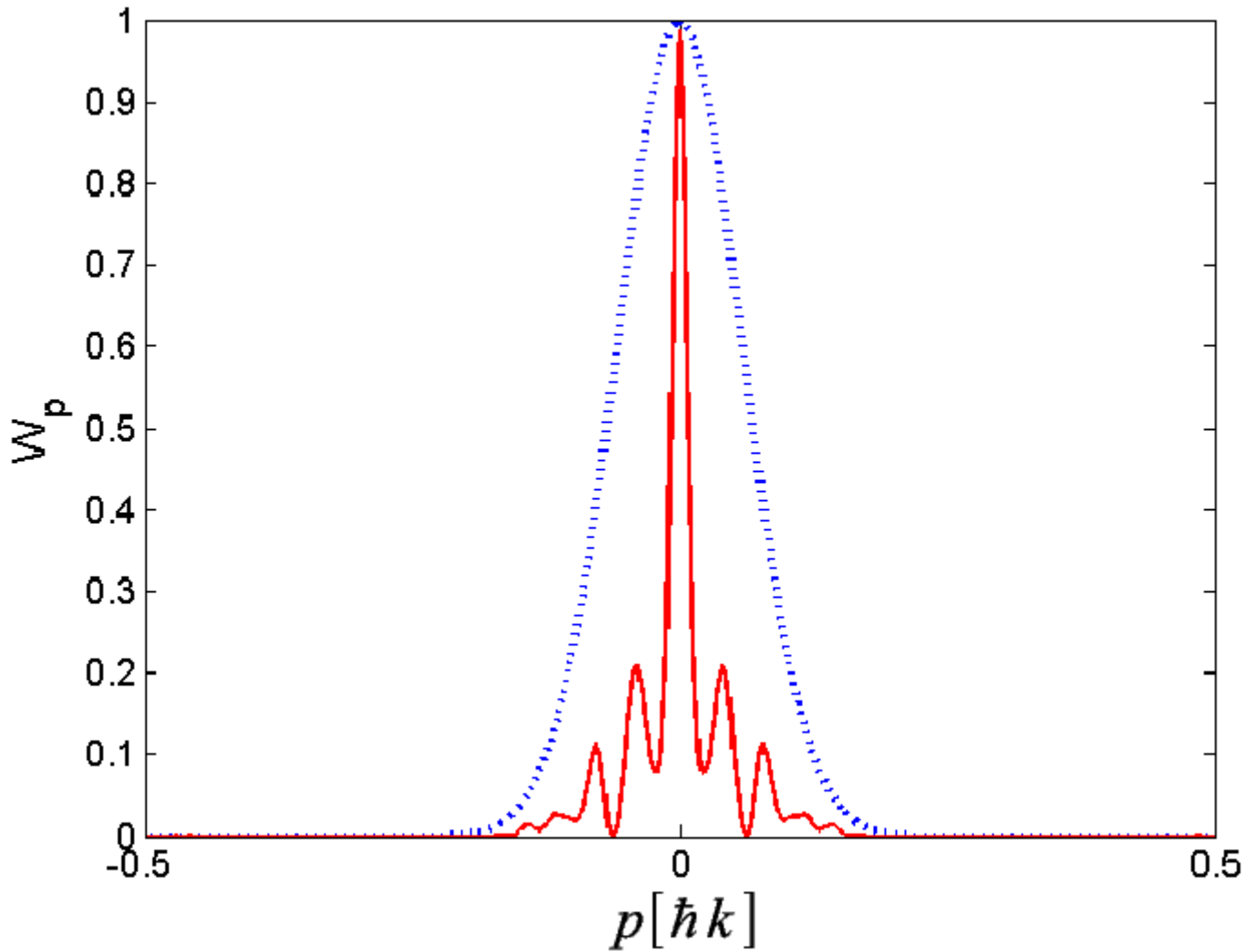


# Time reversal

- System is classically chaotic
- Quantum motion
- Should be reversible!
- Apply number of kicks,  $8\omega_R T = 4\pi + \epsilon$
- Wait for sign change
- Reverse evolution by applying more kicks with  $8\omega_R T = 4\pi - \epsilon$

# Time reversal

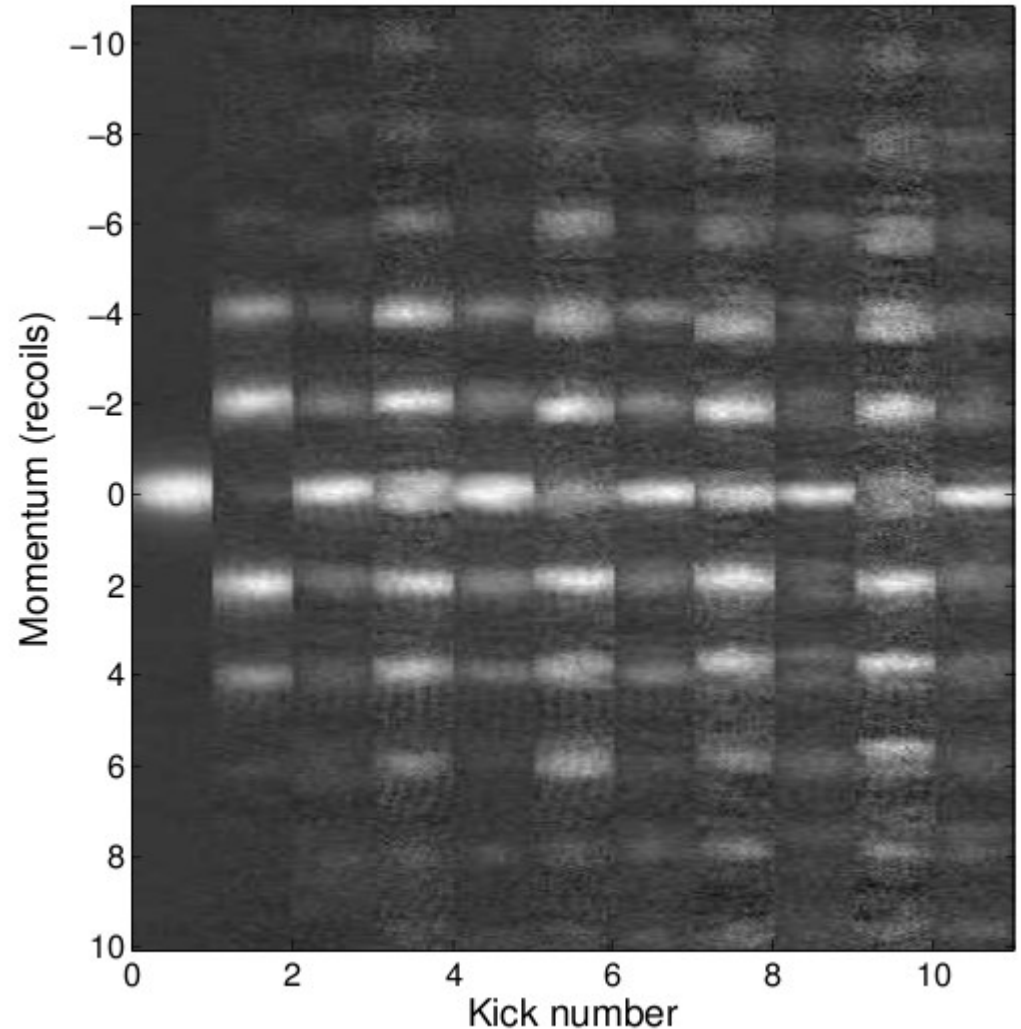
- System
- chaotic
- Quantum
- Shrodinger
- reversal



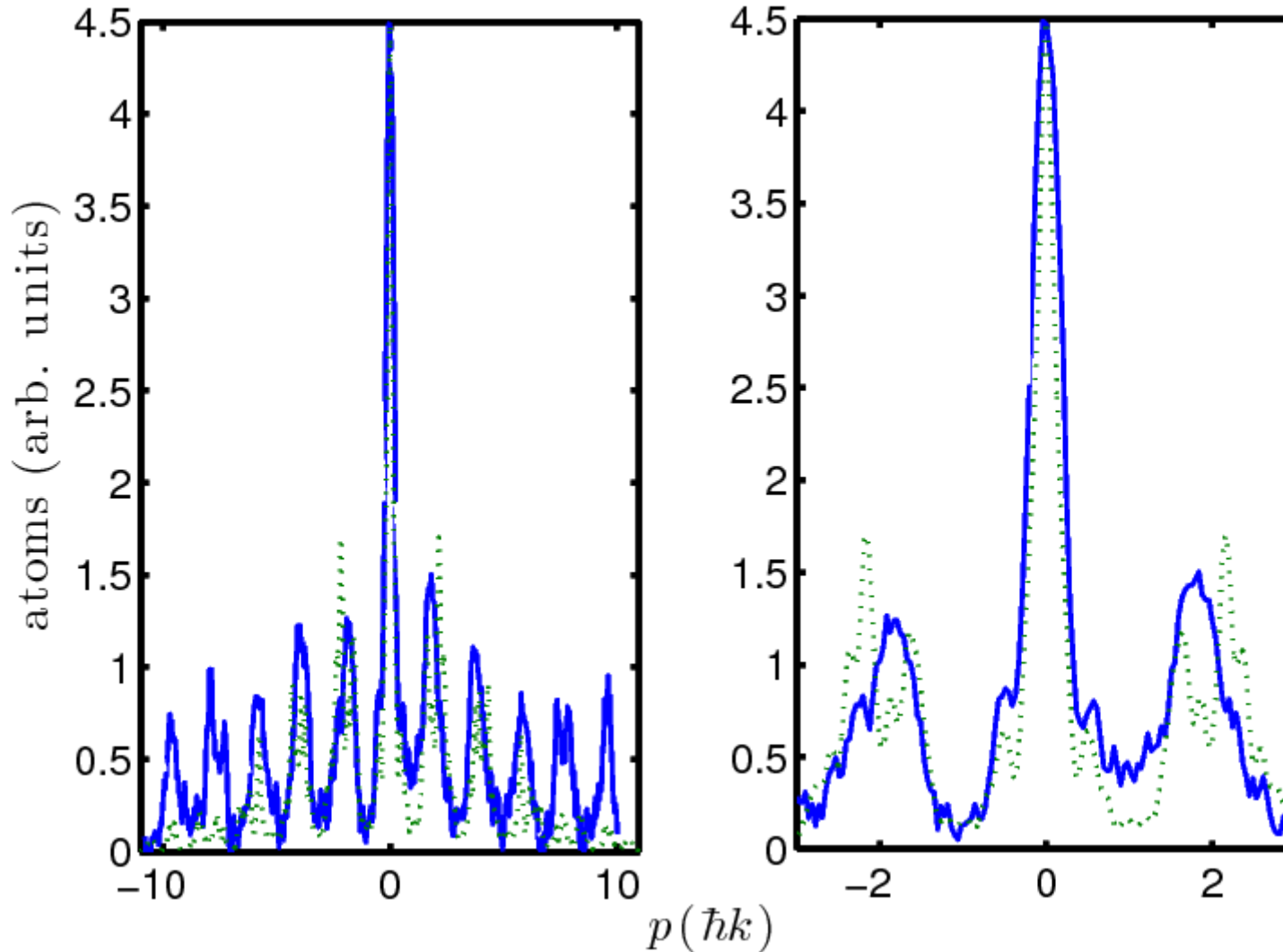
$f$   
 $+\epsilon$   
range  
on  
e

# Time reversal

- Central peak should reach same height after pulse sequence
- Central peak more narrow after pulse sequence

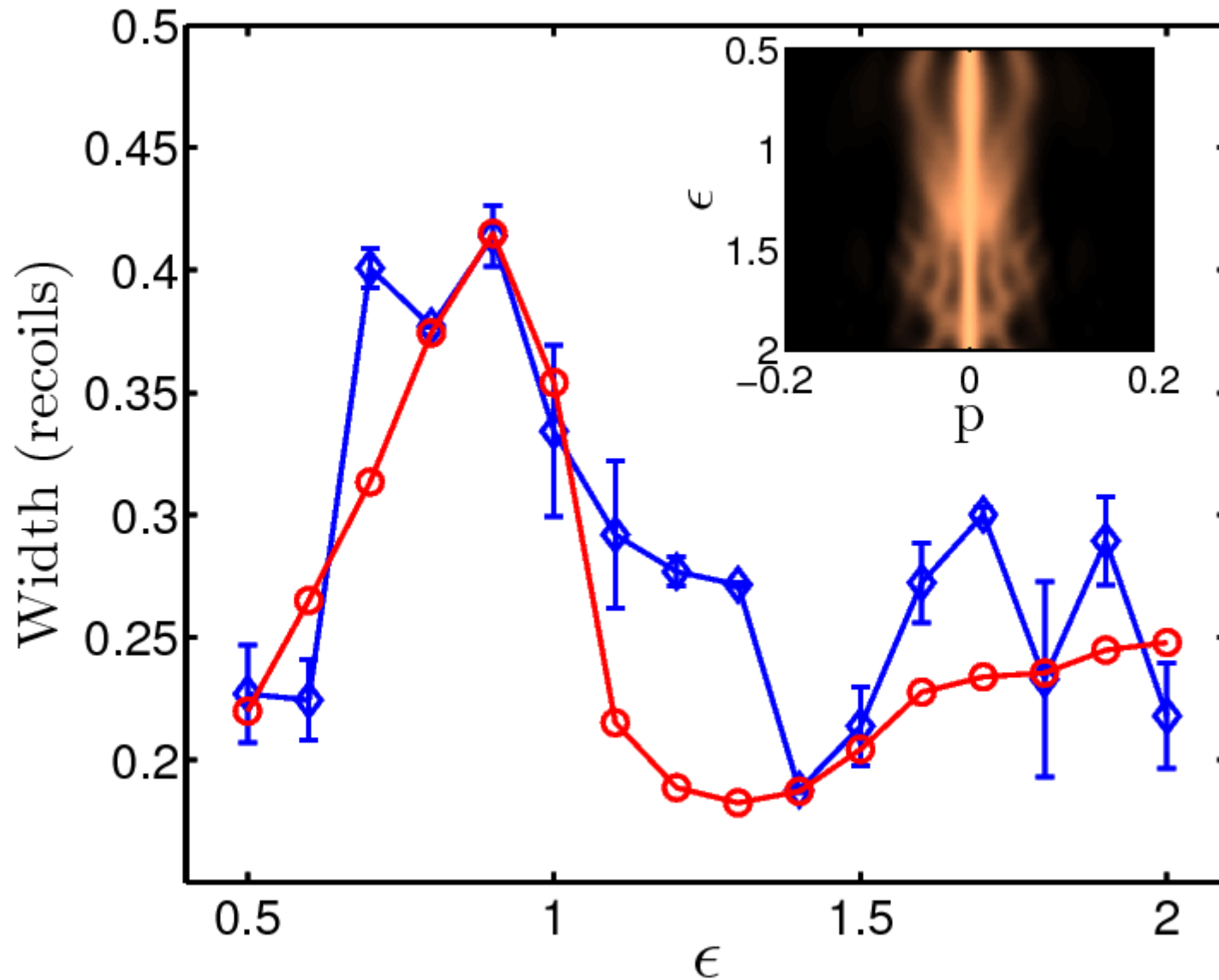


# Momentum distribution





# Changing $\epsilon$



# Future 1: time reversal

- In the ballistic regime, on resonance
  - First apply  $N$  kicks with strength  $\phi_d$
  - Wait  $T_T/2$
  - Wave function reverses sign
  - Apply one kick with strength  $N\phi_d$

# Future 2 : nonlinearity

$$H = \frac{p^2}{2m} + V_0 \cos(2k_l x) \sum_n f(t - nT) + g|\psi^2|$$

PRL **101**, 074102 (2008)

PHYSICAL REVIEW LETTERS

week ending  
15 AUGUST 2008

## Time Reversal of Bose-Einstein Condensates

J. Martin, B. Georgeot, and D. L. Shepelyansky

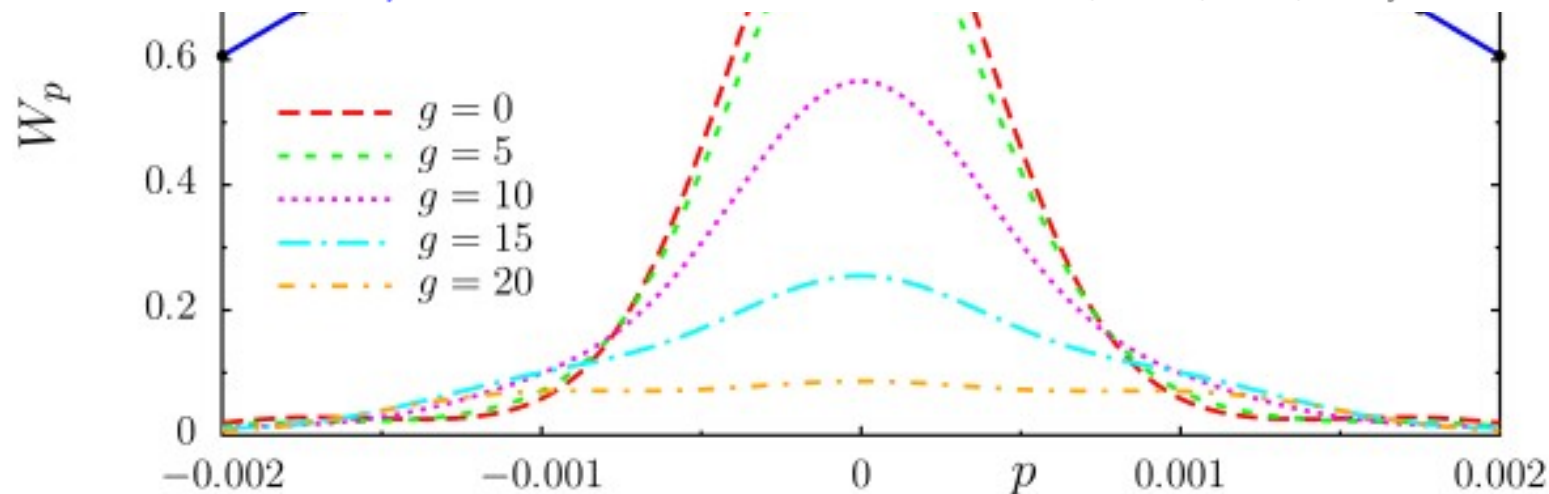
*Laboratoire de Physique Théorique, Université de Toulouse III, CNRS, 31062 Toulouse, France*

(Received 22 April 2008; published 13 August 2008)

Using Gross-Pitaevskii equation, we study the time reversibility of Bose-Einstein condensates (BEC) in kicked optical lattices, showing that in the regime of quantum chaos, the dynamics can be inverted from explosion to collapse. The accuracy of time reversal decreases with the increase of atom interactions in BEC, until it is completely lost. Surprisingly, quantum chaos helps to restore time reversibility. These predictions can be tested with existing experimental setups.

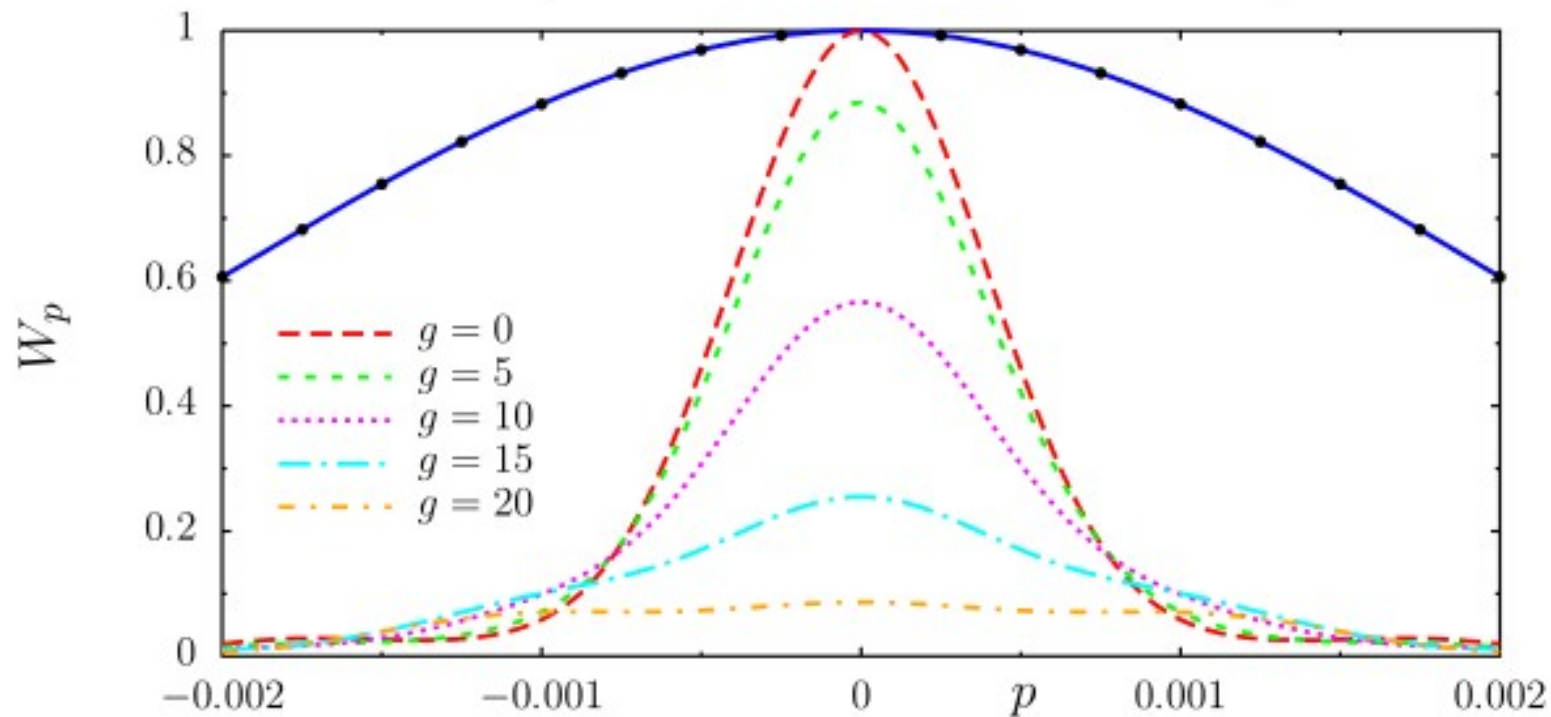
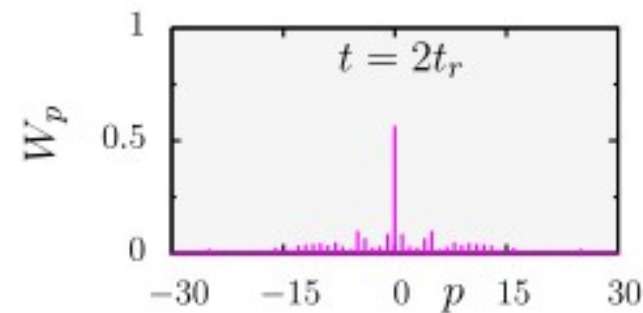
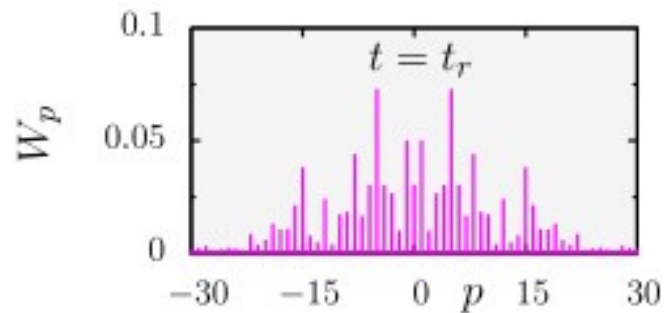
DOI: [10.1103/PhysRevLett.101.074102](https://doi.org/10.1103/PhysRevLett.101.074102)

PACS numbers: 05.45.Mt, 03.75.-b, 37.10.Jk, 67.85.Hj



# Future 2 : nonlinearity

$$H = \frac{p^2}{2m} + V_0 \cos(2k_l x) \sum_n f(t - nT) + g|\psi^2|$$



# Future 3: $\delta$ -kicked harmonic oscillator

PHYSICAL REVIEW A **80**, 023414 (2009)

## Quantum resonances in an atom-optical $\delta$ -kicked harmonic oscillator

T. P. Billam and S. A. Gardiner

*Department of Physics, Durham University, Durham DH1 3LE, United Kingdom*

(Received 25 September 2008; revised manuscript received 22 May 2009; published 20 August 2009)

Under certain conditions, the quantum  $\delta$ -kicked harmonic oscillator displays quantum resonances. We consider an atom-optical realization of the  $\delta$ -kicked harmonic oscillator and present a theoretical discussion of the quantum resonances that could be observed in such a system. Having outlined our model of the physical system we derive the values at which quantum resonances occur and relate these to potential experimental parameters. We discuss the observable effects of the quantum resonances using the results of numerical simulations. We develop a physical explanation for the quantum resonances based on symmetries shared between the classical phase space and the quantum-mechanical time evolution operator. We explore the evolution of coherent states in the system by reformulating the dynamics in terms of a mapping over an infinite two-dimensional set of coefficients from which we derive an analytic expression for the evolution of a coherent state at quantum resonance.

DOI: [10.1103/PhysRevA.80.023414](https://doi.org/10.1103/PhysRevA.80.023414)

PACS number(s): 37.10.Vz, 05.45.Mt, 03.75.Be

# Concluding

- Quantum chaos can be reversed
- Quantum resonances
  - Integer
  - Fractional
  - Velocity dependence
- Simple system
  - More degrees of freedom makes it more classical